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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM. 8

ANALYTICAL METHODS FOR COMPUTING THE POLAR CURVES OF AIRPLANES.

By

Le Sueur.

Translated from "L'Aerophile, August, 1920.
By
Paris Office, N.A.C.A.

March, 1921.

ANALYTICAL METHODS
FOR COMPUTING THE POLAR CURVES OF AIRPLANES.

NOTATION. - The polar curve is in Cartesian coordinates, the curve representing the unit thrust y in terms of the unit drag x.

If we express y and x in a parametrical form in terms of the incidence α, we note that within the usual limits of utilization, y is practically a linear function and x a parabolical function of α.

The following computations will be made with this approximation.

In the "Aerophile" of May 15, 1920, Mr. Rateau assumed for the complete airplane equations in the form:

$$(1) \qquad \begin{cases} Y = Y_o(1 + \eta\, \alpha) \\ X = X_o(1 + \xi\, \alpha^2) \end{cases}$$

and defined a "typical airplane" for which

$Y_o = 0.5897$ $\qquad \eta = 0.3$

$X_o = 0.1204$ $\qquad \xi = 0.017$

It seems to us interesting to give a method by which these equations can be established rapidly, taking as a basis profile tests made in the laboratories.

We must, however, assume units differing from those of Mr. Rateau.

1st. As units of Lift and Drift we take y and x, the coefficients which the Germans call respectively C_a and C_w, which are 1,600 times greater than the Eiffel K_y and K_x.

For that, it is sufficient to correct, for each value of lift, y, the measured values of x and α.

These corrections represent induced drag and deviation, and are given by Munk's formulas.

The profile polar curve is then considered as identical to a parabola within the usual limits of lift, 20 to 120, (corresponding to the Eiffel K_y values varying from 0.0125 to 0.075).

(3) $$x = x_0 + \lambda (y - y_0)^2$$

In the same way, lift in terms of the incidence, is compared to a straight line:

(4) $$\alpha = \beta + \rho y$$

β being the angle of zero lift.

The Göttingen laboratory has made the computations for the above 5 coefficients for 200 different profiles and has drawn up a summary in the form of a Table, completed by two synoptical graphs. (Technische Berichte, II, 3, p.456.)

As an example, we give a few profiles from this Table, corresponding to known planes. (See Table I, next page.

W is the camber of the dorsal profile, D the maximum thickness, and H the thickness at 2/3 of the chord.

These three numbers are given in % of the chord.

TABLE I

No. of Prof.	Specification	Characteristics $x=x_0+\lambda(y-y_0)^2$						$\alpha=\beta+\rho\, y$	
		W	D	H	y_0	x_0	10000λ	β	100ρ
100	Sopwith	7.8	6.7	5.4	45.2	1.50	4.5	-3.2	9.3
125	Hanspeter	13.1	13.0	5.8	78.8	1.80	3.2	-8.2	9.8
126	Gerhardt	7.3	7.1	3.4	46.8	1.26	11.3	-4.7	14.4
256	Junkers E	16.3	16.3	11.8	50.6	2.16	5.4	-7.0	10.2
257	Ago C IV	9.5	8.5	6.0	57.7	2.03	5.0	-4.7	10.0
258	Hydro H Bbg	10.2	5.8	3.5	88.3	2.54	6.5	-5.6	9.7
259	AEG CIV	8.2	6.7	3.2	48.9	1.95	5.3	-2.8	10.5
260	Voisin	9.2	4.1	2.9	91.3	2.67	6.1	-6.0	8.7
264	Fr'hafen 53 RI	10.0	6.5	4.4	72.7	2.28	6.2	-6.2	10.8
265	" 41 RI	11.4	7.4	3.5	89.4	2.86	6.5	-6.4	9.7
268	Rumpler CIV (upper wing)	8.8	6.0	3.6	60.2	2.07	5.6	-4.9	10.7
269	Rumpler CIV (lower wing)	9.2	6.6	4.0	70.4	2.08	7.5	-5.8	11.2
298	Fokker Dr 1	13.7	12.6	7.5	73.8	2.23	4.4	-6.9	10.2
306	MVA Spad	6.8	5.4	4.4	48.1	1.35	7.3	-3.1	8.7

POLAR CURVE OF ANY MONOPLANE WING.

Induced drag, which is added to that of the profile, varies as the square of the lift.

In the same way, induced deviation is proportional to lift.

By the formulas of Betz we may assume:

$$(5)\qquad \begin{cases} \Delta x = \mu y^2 \\ \Delta \alpha = \mu' y \end{cases}$$

μ and μ' are given by Betz in terms of the area S, of the span b and of π.

$$(6)\qquad \begin{cases} \mu = \dfrac{1}{100} \dfrac{S}{(K\,b)^2} \\[2ex] \mu' = \dfrac{1}{100} \dfrac{S}{(K'b)^2} \end{cases}$$

K and K' are coefficients $\leqq 1$ which depend on the geometrical form of the wing.

For rectangular wings $K = K' = 1$.

In these conditions, the polar curve of the monoplane wing becomes

$$(7)\qquad \begin{cases} x = (\lambda + \mu)\left(y - \dfrac{\lambda y_0}{\lambda + \mu}\right)^2 + x_0 + \dfrac{\lambda \mu}{\lambda + \mu} y_0^2 \\[2ex] \alpha = (\rho + \mu')\,y + \beta \end{cases}$$

EXAMPLE. - For a rectangular wing of Aspect Ratio $\dfrac{S}{b^2} = \dfrac{1}{6}$ equations (6) give

$$\mu = 0.00053 \qquad \mu' = 0.03$$

For profile No. 100 (Sopwith wing):

$$(8)\quad \begin{cases} x = 0.00098\ (y - 20.8)^2 + 1.998 \\ \alpha = 0.123\ y - 3.2 \end{cases}$$

It is interesting to compare the figures given by these formulas with those of the laboratories.

1st. COMPARISON WITH GÖTTINGEN.

α	y computed	y measured	x computed	x measured
0	26	25.3	2.0	2.1
1.5	38.2	39.5	2.3	2.3
3	56.4	52.2	3.0	3.1
4.5	62.6	62.7	3.7	3.9
6	74.8	74.7	4.8	4.8
9	99.2	93.6	7.2	7.3

We see that the approximation obtained is of the order of the measurements.

The parabolic formula is thus justified.

2nd. COMPARISON WITH THE EIFFEL LABORATORY.

Wing R A 40 of the Eiffel Laboratory is a Sopwith with an aspect ratio of 6.

The test evaluated in coordinates x and y, gave:

	0°	5°	10°
y	23.8	58.6	87.4
x	1.86	4.02	8.42

For these same values of y, the values of x computed by formula (8) would be respectively

1.99 3.40 6.35

The test results of the two laboratories are therefore not in agreement.

The aspect ratio being the same, we think that these divergences may be attributed to a difference of thickness in the profiles.

We have, in fact, for W, D, and H

Eiffel Profile: 7.3 - 6.4 - 5.5

Göttingen Profile: 7.8 - 6.7 - 5.5

The German profile being thicker, its fineness ratio is less for small angles and greater for large ones.

We had the idea of computing the characteristic coefficients of the polar curve of the Eiffel profile, and found the following values:

$y_o = 3$ $\quad\beta = -3.8$

$x_o = 1.51$ $\quad\rho = 0.125$

$\lambda = 0.009$

The equation of the Eiffel model becomes:

$$(9) \qquad \begin{cases} x = 0.00143\,(y - 19.5)^2 + 1.83 \\ \alpha = 0.155\,y - 3.8 \end{cases}$$

This formula is the one we shall adopt in the following examples.

THE POLAR CURVE OF A CELLULE.

There are several German methods for computing the induced resistances of a cellule based on the profile resistance of a monoplane wing.

We will apply these methods one after the other to a cellule tested at the Eiffel Laboratory:

Cellule RB9, Sopwith profile composed of two rectangular and equal wings measuring 900 x 150.

Gap = 160. Stagger (rear) = 40.

A) MUNK'S METHOD. - This method generalizes the formulas of Betz (5) and (6).

$$(10) \qquad \begin{cases} \Delta x = \mu y^2 \\ \Delta\alpha = \mu' + \mu'' \end{cases}$$

μ and μ' have the same form.

The coefficients K K' and μ'' are given by Munk's diagrams in terms:

1st. Of the ratio $\frac{b}{h}$ of span to gap.

2nd. Of the ratio $\frac{e}{t}$ of depth of stagger to depth of wing.

3rd. Of the angle ϵ of difference between the angles of attack.

Here:

$$\frac{b'}{h} = \frac{900}{160} = 5.6$$

$$\frac{e}{t} = \frac{-40}{150} = -0.266$$

$$\epsilon = 0$$

$$K = 1.1$$

$$K' = 1.05$$

$$\begin{cases} \mu = 0.0083 \\ \mu' = 0.055 \\ \mu'' = 0.4 \end{cases}$$

When all the computations are made, the equation of the polar curve becomes:

$$(11) \qquad \begin{cases} x = 0.00176\ (y - 15.8)^2 + 1.93 \\ \alpha = 0.18\ y - 3.4 \end{cases}$$

B) PRANDTL'S METHOD. - In this method no account is taken of stagger.

In terms of $\frac{h}{b} = 0.178$

we have, in the case of two equal wings, the coefficient of induction

$$\alpha = \frac{1}{1 + 5.3\,\frac{h}{b}} = 0.515$$

which corresponds to a K of Munk equal to

$$K = \sqrt{\frac{2}{1 + \alpha}} = 1.15$$

(instead of 1.11 as before).

We have $\mu = 0.0008$

The equation of the polar curve becomes:

$$(12) \qquad x = 0.0017\ (y - 16.4)^2 + 1.92.$$

C) FORMULAS OF BLASIUS AND HAMBURGER. - These are more precise in the sense that they make for each angle the correction of Drift and the correction of Lift due to interaction.

In the particular case we are concerned with, the two planes have the same dimensions and the same profile.

The formulas are considerably simplified and give:

$$(13)\qquad \begin{cases} \Delta y = \dfrac{t}{2\ h} (\varphi\ ao + \varphi au)\ y \\ \\ \Delta x = \dfrac{t}{2\ h} (\varphi\ wo + \varphi\ wu)\ x \end{cases}$$

The 4 functions φ are given by Tables and graphs in terms of the incidence and of the angle of stagger β. ("Technische Berichte, II, 2, p.341).

We have $\dfrac{t}{2\ h} = \dfrac{150}{2 \times 180} = 0.47$

α	0	5	10
φ ao	-0.13	-0.11	-0.66
φ au	-0.09	-0.14	-0.20
φ wo	+0.013	+0.018	+0.024
φ wu	-0.007	-0.010	-0.017

The values of x and y for the monoplane wing are computed by formula (9) for the Eiffel Laboratory profile.

COMPARISON OF RESULTS. - In Table II we compare the figures given by each of our three computations with those measured by the Eiffel balance.

TABLE II.

		MUNK	PRANDTL	BLASIUS	EIFFEL TEST
$\alpha = 0$	y	18.9	18.9	21.2	20.3
	x	1.9	1.9	1.9	1.9
$\alpha = 5$	y	46.6	46.6	51.7	49.1
	x	3.6	3.5	4.0	3.9
$\alpha = 10$	y	74.4	74.4	76.7	74.4
	x	7.6	7.6	8.4	8.1

We see that these formulas are satisfactory for computing the parabolic polar curve of a rough draft. Munk's method is rather to be recommended when there is a good deal of stagger and much difference between the angles of attack of the two planes of the cellule. Prandtl's method is very suitable for straight cellules of unequal span.

The method of Blasius is rather longer, but more exact, and takes into account the coefficients φ of quantities neglected by Munk and Prandtl. This method is more suitable for a thorough study of a chosen cellule.

Finally, we see by the example of our Sopwith wing, that a slight difference of dimensions in the profile may cause much greater divergences than those occurring between the different methods.

POLAR CURVE OF AN AIRPLANE.

To have the equation for the complete airplane, we have only to add together the drag of the cellule and the drag

caused by structural resistances (landing chassis, rigging, bracing, empennages).

In theory, this drag should be separated into:

1st. A profile drag which will vary, though very slightly, with the incidence.

2nd. An induced drag due to the interaction of all these organs, with respect to each other and also with respect to the wing.

Practically we may combine all these different kinds of drag in one constant coefficient.

This approximation is all the more justified as what we are especially seeking to establish here is a typical equation on which we shall try successively all the combinations studied in a rough draft.

The tested values will, therefore, come in only in a relative fashion.

We thus assume as parabolic equation of the glider:

$$(14) \qquad \left\{ \begin{aligned} x &= (\lambda + \mu) y^2 - 2\lambda y_o y + x_o + \lambda y_o^2 + \upsilon \\ \alpha &= (\rho + \mu') y + \beta + \mu'' \end{aligned} \right.$$

MINIMUM DRAG. - This occurs for lift:

$$Y_o = \frac{\lambda}{\lambda + \mu} y_o$$

and its value is

$$(15) \qquad X_o = x_o + \upsilon + \frac{\lambda \mu}{\lambda + \mu} y_o^2$$

The parabola of the airplane may be put in the form

$$(16) \qquad x = (\lambda + \mu)(y - Y_o)^2 + X_o$$

an equation of the same form as (3).

For instance, if we utilize an airplane whose lift is just Y_0, we see that maximum speed will be realized for X_0 minimum.

a) We reduce x_0 by taking a thinner wing.

b) We reduce υ by diminishing structural resistances (fuselages, rigging, bracing wires).

c) We reduce the last term, which may be written $\mu\, y_0 Y_0$ by diminishing induced resistances. (Action of the gap, aspect ratio and stagger.)

In reality, the requirements of the strength of materials necessitate a compromise which must be solved by tentative methods, and the problem is always brought back to a question of balance.

FINENESS RATIO. - Maximum fineness ratio occurs for the lift

$$Y_1 = \sqrt{\frac{x_0 + \lambda\, y_0^2 + \upsilon}{\lambda + \mu}}$$

and its value is

(17) $$f = \frac{X_1}{Y_1} = 2(\lambda + \mu)Y_1 - 2\lambda\, y_0$$

On these formulas we can study direct the effect of the variation of one of the characteristic coefficients.

MINIMUM POWER. - This is obtained by nullifying the derivate with respect to y in $\frac{x}{y^{3/2}}$. It occurs for

(18) $$Y_2 = -Y_0 + \sqrt{Y_0^2 + 3\,Y_1^2}$$

We have thus a simple relation between the three values of lift corresponding to minimum drag, maximum fineness ratio, and minimum power.

This equation is homogeneous in y.

We shall therefore have an analogous equation in α.

In any airplane whatever, call:

β the angle of zero thrust.

α_0 the angle of minimum drag.

α_1 the optimum angle,

α_2 the angle of minimum power.

These four angles verify the simple relation

$$(19) \quad (\alpha_2 - \beta)^2 + 2(\alpha_0 - \beta)(\alpha_2 - \beta) - 3(\alpha_1 - \beta)^2 = 0$$

independent of the origin chosen for the angles, and containing no coefficient of the airplane.

EXAMPLE. Take again Mr. Rateau's typical airplane with its conventional sign: $\alpha_0 = 0$

Assume, as known: $\beta = -3.33^\circ$ and $\alpha_1 = 5.03^\circ$.

The angle of minimum power, root of equation (19) gives us $\alpha_2 = 8.2^\circ$ as computed by Mr. Rateau.

APPLICATION OF THE METHOD.

Suppose we have an airplane having the cellule previously computed (equation 11) and structural resistance of 0.0016 kg. per m/sec. and per square meter.

The coefficient υ is 1.600 times greater. We have successively the following results:

CHARACTERISTIC COEFFICIENTS OF THE AIRPLANE.

$y_0 = 31$	$\mu = 0.00086$	$\beta = -3.8$
$x_0 = 1.51$	$\mu' = 0.055$	$\rho = 0.125$
$\lambda = 0.0009$	$\nu = 2.6$	$\mu'' = 0.4$

EQUATIONS OF THE POLAR CURVE

$$x = 0.00176\, y^2 - 0.0558\, y + 4.97$$

$$\alpha = 0.18\, y - 3.4$$

MINIMUM DRAG.

$$\begin{cases} Y_0 = 15.85 \\ X_0 = 4.53 \end{cases} \qquad \alpha_0 = -0.55^\circ$$

The equation of the polar curve is written:

$$x = 0.00176(y - 15.85)^2 + 4.53$$

MAXIMUM FINENESS RATIO.

$$Y_1 = 53.2 \qquad \alpha_1 = 6.18^\circ$$

$$f = 0.13$$

MINIMUM POWER.

$$Y_2 = 77.8 \qquad \alpha_2 = 10.6^\circ$$

APPLICATION TO A CONCRETE CASE. - What engine will enable this airplane with 50 square meters of aerofoil to carry 1,500 kg. and fly at 173 km/hr. at an altitude of 3,000 m.?

In these conditions: Weight per square meter = 30.

Static Pressure: q = 105.

We must have a lift: $y = 100 \frac{30}{105} = 28.6$

The equation of the polar curve gives x = 4.8

Total Drag:

$$R = \frac{X}{100} Sq. = \frac{4.8}{100} \times 50 \times 105 = 252 \text{ kg.}$$

Work per second: 252 x 42 = 12,200 kilogrameters.

Assume a propeller efficiency 0.75

The ratio of density = 0.74

The nominal power required is

$$T_o = \frac{12200}{75 \times 0.75 \times 0.74} = 300 \text{ horsepower.}$$

CONCLUSION.

This method is at least as precise as graphical methods.

It is more rapid.

Also, by this method all laboratory results can be carried in a pocket book in the form of Tables of Coefficients.

Knowing the wind tunnel test of a wing and the performances of an airplane of the same profile, it is easy to verify the characteristic coefficients and, at the same time, the methods determining induced resistances. If the results had not been previously checked, this method, entirely analytical, would enable us to discover the error.

It is sufficient to vary the different data successively in close experimentation and to study their influence on each of the characteristic coefficients.

Another important conclusion may be drawn from this article.

All French and foreign laboratories have already carried out many systematic tests.

But results cannot be compared, because comparative calibration has never been attempted.

It would, however, be very simple to build two or three typical models of wings and cellules and to have such standard models in circulation amongst the various wind tunnels throughout the world.

Only when that is done will experimenters everywhere be able to understand each other.